A Simple Equivalent Circuit Model of Parallel Conducting Strip Arrays

Wenting Li, Long Zhang, Yejun He, Sai-Wai Wong, Hancheng Tu
Guangdong Engineering Research Center of Base Station Antennas and Propagation
Shenzhen Key Laboratory of Antennas and Propagation
College of Electronics and Information Engineering, Shenzhen University, 518060, China
Email: w.li@szu.edu.cn, longzhang717@163, heyejun@126.com, wongsaiwai@ieee.org, 2651237146@qq.com

2020 IEEE MTT-S International Conference on Numerical Electromagnetic and Multiphysics Modeling and Optimization (NEMO) | 978-1-7281-6966-8/20/$31.00 ©2020 IEEE | DOI: 10.1109/NEMO49486.2020.9343525

***Abstract*—In this paper, a simple equivalent circuit of parallel conducting strip arrays is proposed. The equivalent circuit is a hybrid circuit which consists of a parallel LC circuit and a series inductor. Compared with reported equivalent circuits, the equivalent circuit presented is quite simple, and it is more accurate when the distance between two adjacent strips is close to the operating wavelength. A strip array is simulated in the commercial software and analyzed through the equivalent circuit method. The properties of the equivalent circuit agree well with those of the strip array, which proves the effectiveness and accuracy of the equivalent circuit model.**

Keywords*—*equivalent circuit, parallel strips, equivalent impedance

I. Introduction

The equivalent circuit method is a quick and simple approach to study antenna arrays, frequency selective surfaces, artificial high impedance surfaces, metamaterials and other periodic structures. Compared with simulating the periodic structures in commercial software directly, investigating their equivalent circuits could reveal some characteristics of the periodic structures quickly. Although some equivalent circuits of periodic structures in free space are given [1], the models are valid with restrictions. Periodic metal grids and plates are investigated in [2]. A model for lossy planar periodic structures at low frequencies is presented in [3]. However, the models in [2] and [3] are quite complex, which makes it inconvenient to build an equivalent circuit of strip arrays.

The parallel strip array is studied in [4], and the analytic expression of the impedance and the equivalent circuit model of the strip array are given as well. In [5], the parallel strip array printed on a substrate is investigated. However, the equivalent circuit models in [4] and [5] are effective under the limitation that the distance between adjacent strips is far less than the working wavelength. With the operating frequency increasing, the equivalent circuit model presented in [4] is not accurate enough.

In this paper, a simple equivalent circuit model of the strip array is proposed. The model is composed of a parallel LC circuit and a series inductor. It describes the properties of the parallel conducting strip array for normally incident plane waves with reasonable accuracy. When the distance between adjacent strips is close to working wavelength, the characteristics of the equivalent circuit still approximate those of the strip array with relatively small error.

II. Equivalent Circuits of Parallel Conducting Strip Arrays

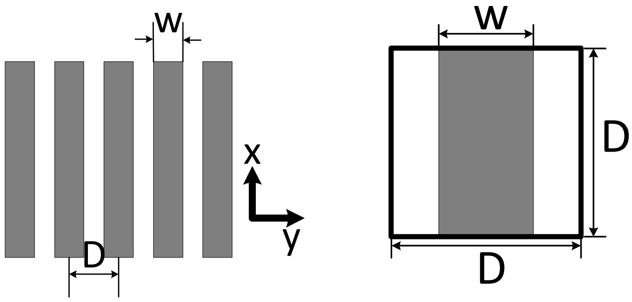
The configuration of the parallel conducting strip array is shown in Fig. 1(a) . Fig. 1(b) shows the unit cell of the array. The array is placed in free space. The distance between two adjacent strips is *D*. The width of the strip is *w*. The strip is along x-axis. When a plane wave with electric field along x-axis illuminates the strip array normally (incidence angle=0), the impedance of the strip array is Z_g, given by (1) in [4], where η is the wave impedance in free space, and α_{ABC} is the grid parameter.

$$Z_g = j \cdot \frac{\eta}{2} \alpha_{ABC} \tag{1}$$

α_{ABC} is given by

$$\alpha_{ABC} = \frac{kD}{\pi} \cdot \left[\ln\left(\frac{1}{\sin\left(\frac{\pi w}{2D}\right)} \right) + \frac{1}{2} \sum_{n=-\infty}^{+\infty} \left(\frac{2\pi}{\sqrt{(2\pi n)^2 - (kD)^2}} - \frac{1}{|n|} \right) \right] (n \neq 0) \tag{2}$$

where *k* is the wave number in free space.



(a) (b)

Fig. 1. The configuration of the parallel conducting strip array (a) and its unit cell (b)

When the equivalent circuit is modeled in [4] and [5], the sum of series in the square bracket in (2) is neglected when $D \ll \lambda$, where λ is the wavelength. In [4], *D* is set to 0.12λ at most. The equivalent circuit in [4] only applies the dense array, of which the distance between adjacent cells is much less than the wavelength. However, for the array which does not satisfy the restriction that $D \ll \lambda$, the equivalent circuit proposed in [4] may result in quite large error.

Fig. 2 shows the imaginary part of the impedance of the strip array. In this instance, *D* is set 40mm, and *w* is 4mm. The imaginary part of the impedance is given when the frequency changes from 1 to 7GHz. When $D/\lambda>0.4$, the error of the equivalent circuit in [4] grows rapidly with frequency increasing. In addition, the calculated results given by (1) agree with the simulated results in HFSS.

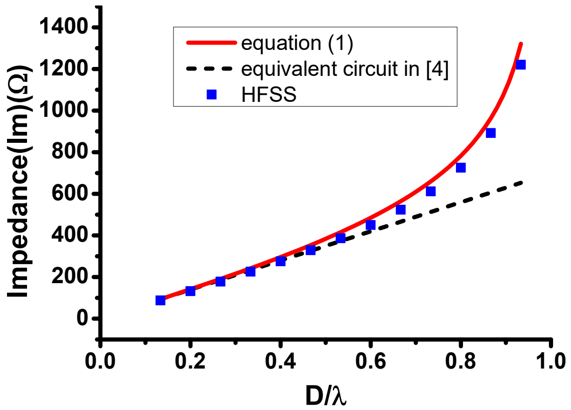


Fig. 2. The real imaginary part of the impedance of the parallel strips

For common arrays, the distance between adjacent cells is less than wavelength to avoid grating lobes. Therefore, in this paper, the equivalent circuit is discussed only when $D \leq \lambda$.

When D is far less than λ, the impedance of the equivalent circuit in [4] Z_g^0 is given by

$$Z_g^0 = j \cdot \frac{kD\eta}{2\pi} \log\left(\frac{1}{\sin\left(\frac{\pi w}{2D}\right)}\right) \tag{3}$$

Therefore, the equivalent circuit is an inductor. As $k = \omega\sqrt{\varepsilon_0\mu_0}$, the value of inductance L_0 is given by

$$L_0 = \frac{\sqrt{\varepsilon_0\mu_0}D\eta}{2\pi} \log\left(\frac{1}{\sin\left(\frac{\pi w}{2D}\right)}\right) \tag{4}$$

When D is not far less than λ, the strip array cannot be treated as a single inductor. It could be equivalent to a hybrid circuit containing a parallel LC circuit and a series inductor approximately, which is shown in Fig. 3.

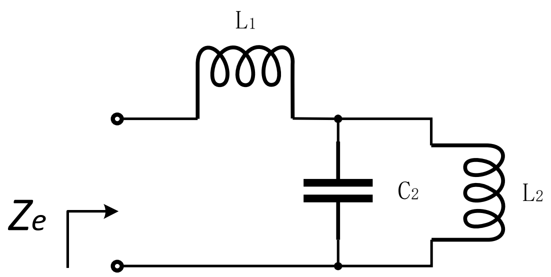


Fig. 3. The equivalent circuit of the strip array

According to the configuration of the hybrid circuit, the impedance of the equivalent circuit Z_e is given by

$$Z_e = j\omega L_1 + \frac{j\omega L_2}{1-\omega^2 L_2 C_2} \tag{5}$$

When $\omega = \omega_0 = \frac{1}{\sqrt{L_2C_2}}$, the equivalent circuit resonates, and Z_e reaches infinity. Observing the sum of series in (2), as $D \leq \lambda$, that $\lambda = \lambda_0 = D$ could lead to the singularity of Z_g as well, if n=±1. Since Z_e is the approximation of Z_g, ω_0 is supposed to equals $2\pi/(\lambda_0\sqrt{\varepsilon_0\mu_0})$. Therefore, L_2 and C_2 should satisfy (6).

$$C_2 = \frac{D^2\varepsilon_0\mu_0}{4\pi^2 L_2} \tag{6}$$

As C_2 is determined by L_2. the equivalent circuit only contains two parameters L_1 and L_2, which makes it quite convenient to build the equivalent circuit of the strip array.

III. Resuts Analysis

In this section, the impedance of the strip array with certain dimensions is calculated according to (1). Then, an equivalent circuit of the strips is presented. In addition, the parallel conducting strips are simulated in HFSS as well.

Fig. 1 shows the strip array, in which D is 40mm and w is 4mm. The parameters of the equivalent circuit are given in Table I.

TABLE I. PARAMETERS OF THE EQUIVALENT CIRCUITS

L_1	L_2	C_2
1.32×10^{-8}(H)	1.80×10^{-9}(H)	2.50×10^{-13}(F)

Three methods are applied to obtain the impedance of the parallel strip array respectively. Fig. 4 shows the real part of the impedance. Equation (1) and the equivalent circuit both reveal that the real part of the impedance is zero while the simulated real part is smaller than 2ohm from 1 to 7GHz.

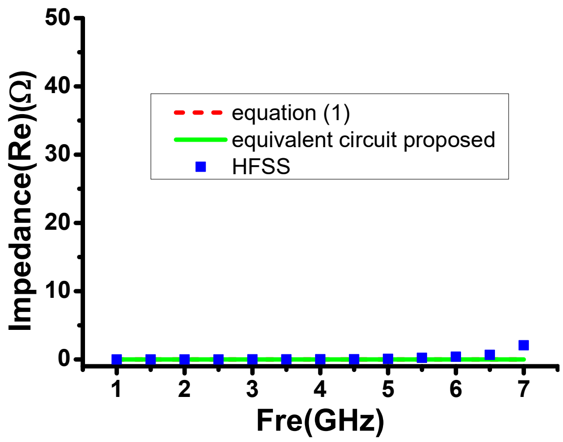


Fig. 4. The real part of the impedance of the strip array

The imaginary part of the impedance is shown in Fig. 5. The results from the equivalent circuit agree well with the simulated results. It can be seen that the calculated results from (1) are slightly larger than the simulated results. The results from the equivalent circuit are larger than the simulated results from 1 to 4.5 GHz while the results from the equivalent circuit are smaller than those in the simulation from 5 to 7 GHz.

Fig. 6 and Fig. 7 show the reflection coefficient and transmission coefficient of the strip array when it is illuminated normally by a plane wave. With the frequency increasing, the reflection coefficient decreases and the transmission coefficient rises. Moreover, in respect of reflection and transmission coefficients, the equivalent circuit approximate the strip array well.

From the comparison above between the strip array and its equivalent circuit, it is practical to predict the characters of the strip array through analyzing its equivalent circuit, which introduces a simple and accurate tool to analyze other similar periodic structures.

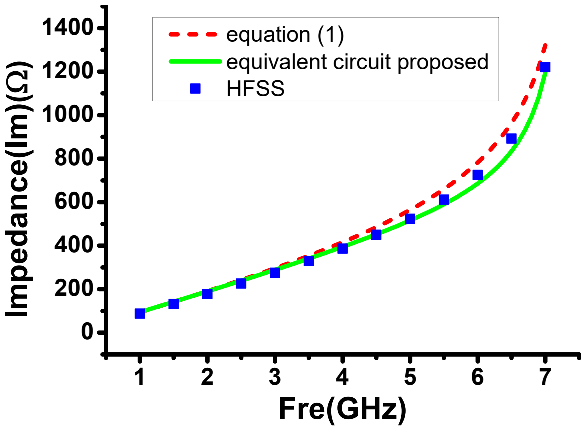


Fig. 5. The imaginary part of the impedance of the strip array (*w/D=0.1*)

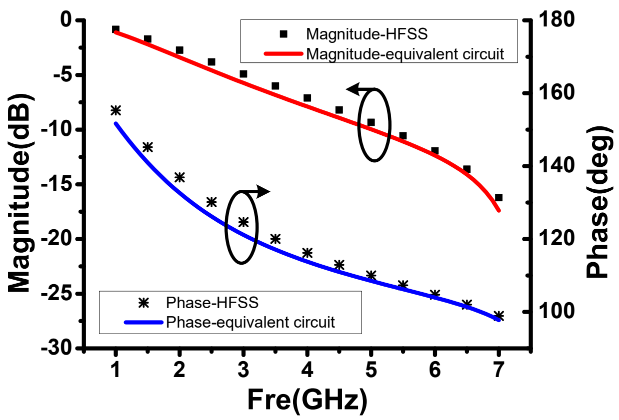


Fig. 6. The reflection coefficient of the strip array (*w/D=0.1*)

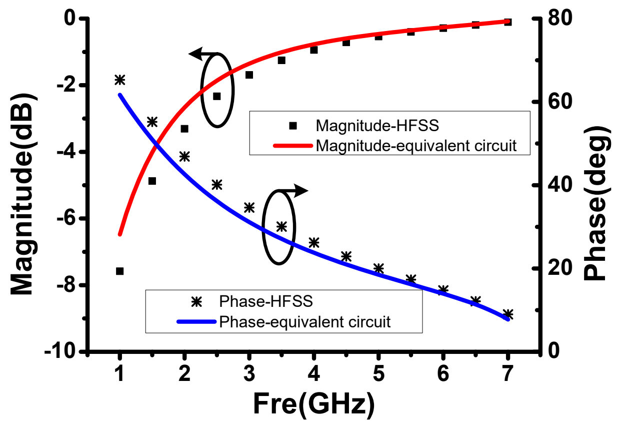


Fig. 7. The transmission coefficient of the strip array (*w/D=0.1*)

The equivalent circuit proposed is based on (1) and (2). Equation (2) is initially used to describe the grid parameter of thin conducting strip arrays. Therefore, the smaller *w/D* is, (2) is more accurate. If *w* is getting close to *D*, the accuracy of the impedance given by (1) deteriorates.

Fig. 6 shows the imaginary part of the impedance of the strip array when *w/D*=0.5. The error of the impedance from equation (1) is larger than that when *w/D*=0.1. As the error of impedance is enlarged, the reflection and transmission coefficients from equation (1) are not accurate enough, which are shown in Fig. 9 and Fig. 10.

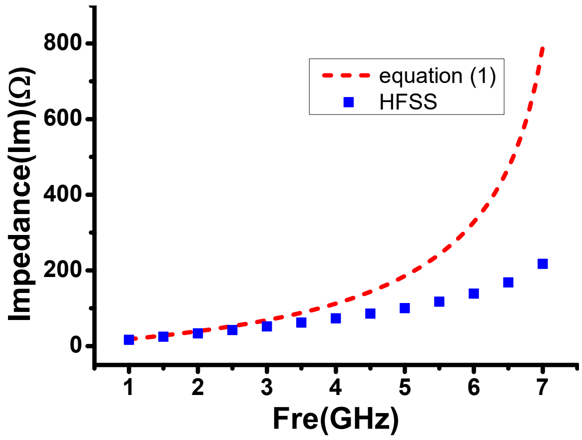


Fig. 8. The imaginary part of the impedance of the strip array (*w/D=0.5*)

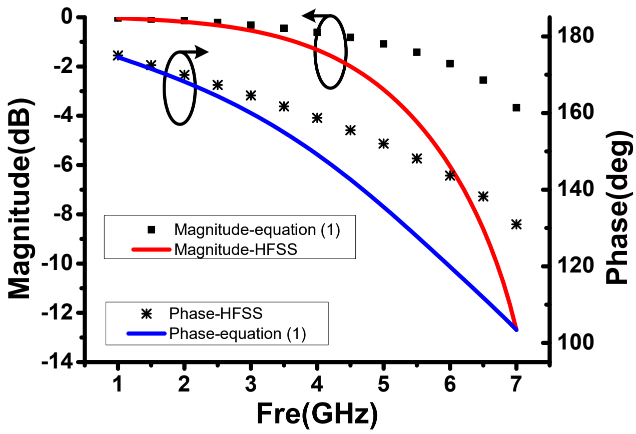


Fig. 9. The reflection coefficient of the strip array (*w/D=0.5*)

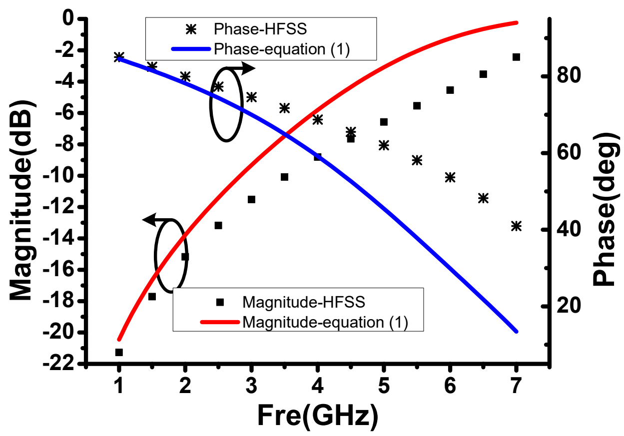


Fig. 10. The transmission coefficient of the strip array (*w/D=0.5*)

Moreover, in the equivalent circuit, the sum of series in the square bracket in (2) is replaced by a parallel LC circuit. This substitute also brings in the differences between the equivalent circuit and (1).

IV. Concusion

In this paper, the strip array is modeled through a simple equivalent circuit. The impedance, reflection and transmission coefficients of the equivalent circuit agree well with those simulated in commercial software. Therefore, the equivalent circuit can be applied as a tool to analyze the properties of other periodic structures which could be decomposed to strip arrays.

V. ACKNOWLEDGMENT

This work was supported in part by the National Natural Science Foundation of China under Grant 62071306.

REFERENCES

[1] N. Marcuvitz, "Waveguide handbook," ed: Iet, 1951, pp. 217-243.

[2] L. Shung-Wu, G. Zarrillo, and L. Chak-Lam, "Simple formulas for transmission through periodic metal grids or plates," IEEE Trans. Antennas Propag., vol. 30, pp. 904-909, 1982.

[3] K. W. Whites and R. Mittra, "An equivalent boundary-condition model for lossy planar periodic structures at low frequencies," IEEE Trans. Antennas Propag., vol. 44, pp. 1617-1629, 1996.

[4] S. Tretyakov, Analytical modeling in applied electromagnetics: Artech House, 2003.

[5] O. Luukkonen, C. Simovski, G. Granet, G. Goussetis, D. Lioubtchenko, A. V. Raisanen, and S. A. Tretyakov, "Simple and Accurate Analytical Model of Planar Grids and High-Impedance Surfaces Comprising Metal Strips or Patches," IEEE Trans. Antennas Propag., vol. 56, pp. 1624-1632, 2008.